

Science Olympiad  
Astronomy C  
National Event - Michigan State University

May 25, 2024



**Directions:**

- Each team will be given **50 minutes** to complete the exam.
- There are three sections: **§A** (Qualitative), **§B** (JS9), and **§C** (Quantitative).
- For each section, use the appropriate Image/Illustration Set at the end of the exam packet.
- For calculation questions, provide **3 significant figures** and **please show your work**.
- The use of AI tools (e.g. ChatGPT) are expressly forbidden.
- The top five tiebreakers, in order, are: Q15, Q13, Q18, Q3, Q16.
- The exam will be posted soon after the competition at <https://chandra.si.edu/edu/>.
- Good luck! And may the stars be with you!

**Written by:**

**The Astronomy A-Team**

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Terry Matilsky, Rio Sessions, Jim Ciccarelli, et al.

## Section A: Qualitative

This section consists of qualitative questions about this year's deep sky objects and core astronomy concepts. Questions refer to Image/Illustration Set A/B. Unless otherwise specified, each question is worth 1 point, for a total of 52 points.

- Image 20 shows objects within the massive star formation complex, the Carina Nebula.
  - What are the dark, opaque features?
  - What is the process that carves out these irregular features?
  - These features continue to condense and erode. What is the image number of the Hubble observation that shows the extreme violence and turbulence associated with these features as they condense into protostars?
- As protostars rapidly collapse, the envelope of material that surrounds them form accretion disks. As protostars transition to stars, the characteristics of these disks change over time, leaving debris disks around the stars.
  - What causes the disk material to move away from the protostar?
  - What image number shows an illustration of this protostar transitive stage?
- The H-R diagram is a plot of physical properties of stars in various evolutionary stages or in the process of transitioning between stages.
  - What are the two major classifications of pre-main sequence protostars?
  - What letters show their locations on the H-R diagram?
  - What process enables these objects to become main sequence stars?
- A close star formation complex exhibits the usual chaotic star forming processes, pre-main sequence objects, emission and reflection nebulas, dark clouds, and astronomical jets.
  - What is the name and image number that shows this star formation region?
  - Image 13 contains a short, transient stage of collapsing protostars in this region. What are these types of objects called and what is the name of these objects in this image?
  - How are these objects formed?
- Brown Dwarfs never become main sequence stars. However, these failed stars are interesting, and sometimes display unexpected properties and activity.
  - What is the name and image number of a WISE observation of 2 orbiting brown dwarfs?
  - [1.5 pts] What are the different weather patterns on these two brown dwarfs, and which image shows the patterns on one of them?
  - What are the letters on the H-R diagram that show the locations of these two orbiting brown dwarfs?
  - What is the name and image number of the brown dwarf that was the first detected to have an exoplanet companion?

6. The ALMA observation in Image 5 is a classical T Tauri star surrounded by a disk. This protostar is still accreting materials from the surrounding gas and dust.
  - (a) What is the name of this protostar and what is the number of the image that suggests two exoplanets are forming within the disk?
  - (b) What is the number of the image that shows the behavior of this class of T Tauri stars?
  - (c) A compound which has been missing from observations of these disks has now been found. What is the compound, and what is the image number that shows its distribution?
7. Early protostars form as a core accreting the surrounding gas and dust until the dynamics of the collapse and radiation bursts from the protostars form protoplanetary disks. Gaps appear in the disk where protoplanets are starting to develop. There is evidence of a second, more violent type of process that was detected in a Herbig Ae protostar that also results in the formation of protoplanets.
  - (a) How are protoplanets created by this process?
  - (b) What is the name and image number of the protostar and forming protoplanets thought to be produced by this second process?
  - (c) What is the number of the image that is a plot of this protostar's behavior?
  - (d) What is the name and image number of a similar type of object forming in this way that is producing compounds such as silicon monosulfide from powerful shockwaves.
8. Two of the WASP exoplanets are illustrated in Images 8 and 17. These exoplanets discovered by WASP are gas giants, composed of a small solid core surrounded by hydrogen and helium. They can be much more massive than Jupiter, and orbit extremely close to their parent stars.
  - (a) [1.5 pts] The NASA illustration of WASP-39b in Image 17 has a mass of 0.28 Jupiters and orbits in 4.1 days. It is the first exoplanet observed by JWST. What two compounds did it detect for the first time in an exoplanet atmosphere? What image number shows its atmospheric spectrum?
  - (b) The WASP-43b exoplanet is tidally locked with its parent star, orbiting every 19 hours. Two different studies produced a detailed weather map. What is the image number of the map, and what method was used to construct the map?
9. Radial velocity, transits, and direct imaging are three methods of detecting exoplanets, and most exoplanets have been detected using the transit method.
  - (a) Why does the transit method work best for gas giants?
  - (b) What is the name and image number of the first gas giant exoplanet orbital motions confirmed by direct imaging?
  - (c) What is the name and image number one of the first debris disks directly imaged around another star?
  - (d) [1.5 pts] The parent stars of the two objects in parts (a) and (b) above are located on what branch of the H-R diagram? What letters show their approximate locations?

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10. X-ray and high energy radiation from energetic newly formed stars strip away any atmosphere or oceans that might start to form on any developing protoplanets. V1298 Tau b, illustrated in Image 7, is a prime example of this situation.
- (a) What is the number of the image that plots the behavior of its parent star?
  - (b) What behaviors are thought to be causing the variations for this star?
11. The gas and dust that surrounds condensing protostars becomes a protoplanetary disk around its parent star. The protoplanetary disk evolves into a debris disk. Image 16 illustrates the basic structure of debris disks.
- (a) What is the fundamental difference between a protoplanetary disk and a debris disk?
  - (b) How can debris disks persist for much of a star's lifetime?
  - (c) [1.5 pts] What three types of large objects reside in debris disks?
12. Terrestrial planets have a central metallic core, surrounded by a silicate mantle with a variety of surface features such as canyons and, mountains and volcanoes.
- (a) What is the name of the central star of the terrestrial planetary system illustrated in Image 14 and what letter shows its location on the H-R diagram?
  - (b) This system has three exoplanets in the habitable zone; however recent studies have revealed that the closest-in planet has been found to have no detectable atmosphere with JWST. What process is destroying the atmosphere?



13. **Feeling Blue.** Light is the keystone of astronomy. Image 27 shows the spectral energy distributions (SEDs) of four main sequence stars at different temperatures approximated as blackbodies.

- (a) [2 pts] Assume that stars A and B are viewed from the same distance. Which star is more luminous? Which star is “redder”? Briefly (1–2 sentences) justify your answers using the provided SEDs.

Rather than measuring the entire SED of a star, which is difficult and time consuming, astronomers often use filters to measure the intensity of a star in two spectral bands. They subtract their magnitudes to find the star’s color index.

- (b) Some examples of bands/filters are labeled with vertical lines in Image 27: B (445 nm), V (550 nm), [3.6] ( $\mu\text{m}$ ), and [4.5] ( $\mu\text{m}$ ). What part(s) of the EM spectrum do these four filters fall under?
- (c) [1.5 pts] Star A is 1 pc away from Earth and has a B–V color index of  $-0.79$ . What would its color index be if it is 2 pc away?

The answer to part (d) demonstrates one of the strengths of using color indices. But astronomers must carefully select which filters they use. The table below shows the effective temperatures, B–V color index, and [3.6]–[4.5] color index of each star.

Star	$T_{\text{eff}}$ [K]	B–V	[3.6]–[4.5]
A	29000	$-0.79$	$-0.95$
B	7500	$-0.24$	$-0.91$
C	4900	$0.22$	$-0.87$
D	2600	$1.43$	$-0.76$

- (d) [1.5 pts] Using its  $T_{\text{eff}}$ , what is the MK classification (spectral type and luminosity class) of star C?
- (e) [1.5 pts] Which color index is the best for predicting temperature? Briefly justify your answer using the table or the SEDs.
- (f) [1.5 pts] In reality, extinction by intervening interstellar dust reddens the light we see from stars. How would this affect the temperatures predicted using color indices?

14. **Seeing Red.** A beautiful snapshot of the Orion Nebula by JWST can be seen on the cover of the exam. It is the first H II region to be discovered.

- (a) What spectral type of stars create H II regions?
- (b) How does this explain the observation that almost all H II regions are found in stellar nurseries?

Within H II regions, the ionizing effect of the star’s radiation (which stays relatively constant) competes against the natural rate of recombination (which is proportional to the size of the H II region).

- (c) What are the two reactants and two products of recombination?
- (d) [1.5 pts] If ultraviolet radiation is required to ionize ground-state hydrogen, why are H II regions red?

Let’s investigate a simple model of the formation of an H II region where a star suddenly pops into existence within a uniform medium of neutral hydrogen gas. The star forms a spherical H II region that eventually grows to its final radius  $R_{\text{final}}$  when ionization and recombination are in equilibrium.

- (e) [1.5 pts] Which curve (A–D) in Image 28 most accurately captures the change of the region’s radius with time? Briefly justify your choice. (*Hint: The rate of recombination is proportional to what?*)

## Section B: JS9

In this section, you will use the JS9 website to analyze a Chandra observation of a red dwarf star. Points are shown for each part, for a total of 15 points.

To display the data:

- Go to <https://chandra.si.edu/js9/>.
- Click on The Unofficial Chandra Archive Search Page button.



- In the first text box (labeled “Chandra Obs ID”), enter the number 20619. Then click “Search”.

The Unofficial [Chandra](#) Archive Search Page

20619  Observer (PI)  Title

RA (hh:mm:ss.s)  Dec (dd:mm:ss.s)  Size (dd

Object Name  Simbad@Cfa  Any  Any

- Next, drag and drop the title link into the JS9 window. **Do not click on the link.**

The image shows the search results page. It says "Found: 1 match". Below that, there is a table with the following columns: ObsID, RA, Dec, Exposure, Observer, and Title. The table has one row with the following data: 20619, 17:57:47.749, 04:44:50.616, 26.7, FRONING, and [MEGA-MUSCLES TREASURY SURVEY: MEA...](#). The title link is circled in red. Below the table, there is a section titled "The Chandra Metadata Table - 26660 records". At the bottom, there is a JS9 window showing a FITS image of the star. A red arrow points from the title link in the table to the JS9 window.

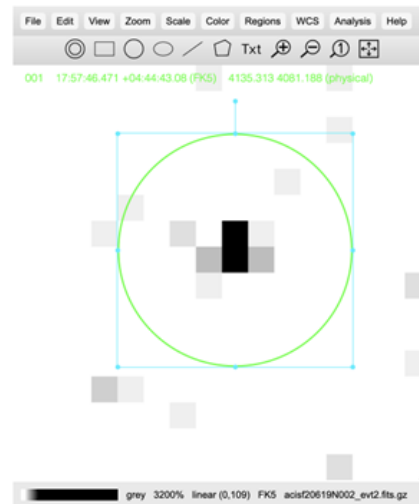
15. First, open the FITS header file to answer the following questions.

- [1 pt] What is the name of this object?
- [1 pt] On what day was this object observed?

Now, take a look at the image. It will be easier to see if you change the scale from linear to logarithmic.

- [1 pt] Briefly describe what feature(s) you see in this image.

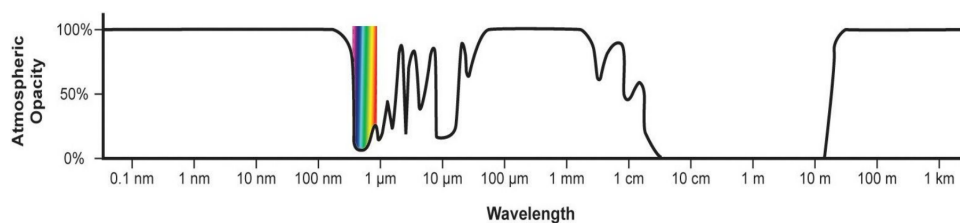
Now, zoom in on the brightest pixels in the center of the image. Place a small region around these, similar to the one shown below. (*Note: The color scheme has been inverted for this image, to save on ink.*)



*Keep this region in your JS9 window for the remainder of the questions!*

Run an energy spectrum on this object, to answer the following questions.

- (d) [1.5 pts] Does this object emit primarily soft X-rays (energies less than around 2000 eV) or hard X-rays (energies greater than around 2000 eV)?
- (e) [1.5 pts] At what approximate energy (in eV) does this object have the strongest X-ray emission at (i.e. highest counts reading)?
- (f) [2 pts] What wavelengths do photons of these energies correspond to, in nm? (*Hint: Use  $h = 6.58 \times 10^{-16}$  eV s for Planck's constant, and  $c = 3.00 \times 10^{17}$  nm/s for the speed of light so that you don't have to perform ugly unit conversions.*)
- (g) [2 pts] Look at the following chart of atmospheric spectral windows for Earth. Would these photons be likely to reach Earth's surface?



Run a light curve analysis on this object. For this question, use the first light curve analysis, under 'Server-Side Analysis', NOT the version under NSO analysis.

- (h) [1.5 pts] At what time did this object flare up, as reported on the x-axis?
- (i) [1.5 pts] What was the height of the peak of this object, in counts, when it flared up?
- (j) [2 pts] How do flares like the one observed here impact planetary systems surrounding red dwarf stars such as this object?

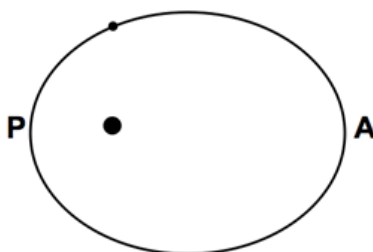
## Section C: Quantitative

This section consists of quantitative questions about two planetary systems. Questions refer to Image/Illustration Set C. Points are shown for each part, for a total of 33 points. Please show your work. Partial credit may be awarded for correct work.

Conversions that may be helpful:

- $1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$  (Ast. unit)
- $1 \text{ ly} = 9.461 \times 10^{15} \text{ m}$  (Light-year)
- $1 \text{ pc} = 3.086 \times 10^{16} \text{ m}$  (Parsec)
- $G_{SC} = 1361 \text{ W/m}^2$  (Solar constant)
- $1 M_{\odot} = 1.989 \times 10^{30} \text{ kg}$  (Solar mass)
- $1 R_{\odot} = 6.957 \times 10^8 \text{ m}$  (Solar radius)
- $1 L_{\odot} = 3.828 \times 10^{26} \text{ W}$  (Solar luminosity)
- $G = 6.674 \times 10^{-11} \text{ m}^3/\text{kg}/\text{s}^2$  (Gravitational constant)

16. **A Two Body Problem.** In 2000, a planet was discovered orbiting a star in the constellation Orion, with a mass of  $1.48 M_{\odot}$ , a radius of  $1 R_{\odot}$ , and a luminosity of  $6.16 L_{\odot}$ . This particular planet has a mass of  $10.4 M_{\text{J}}$ . Its apoastron (A) distance is 5.09 AU and its periastron (P) distance is 2.41 AU. A face-on view of the orbit shown in the image below (not to scale).



The *vis-viva* equation is a key relation that models an orbit of a low-mass, orbiting body around a high-mass, central body:

$$v^2 = GM \left( \frac{2}{r} - \frac{1}{a} \right),$$

where

- $v$  is the velocity of the orbiting body (the planet),<sup>1</sup>
  - $r$  is the distance between the two bodies,
  - $a$  is the semi-major axis of the orbit,
  - $G$  is the gravitational constant,
  - and  $M$  is the mass of the central body (the star).
- (a) [1 pt] The conservation of what property leads to the vis-viva equation?
- (b) [1 pt] Use the vis-viva equation to find the velocity of the planet at apoastron and periastron, in km/s.
- (c) [2 pts] If this system is viewed edge-on from Earth, calculate the range of transit duration, in hours, we **could possibly** observe. Assume that the planet's radius is much smaller than the star's.  
(If you did not answer part (b), use  $v_A = 30 \text{ km/s}$  and  $v_P = 40 \text{ km/s}$ .)
- (d) [2 pts] The Nancy Grace Roman Space Telescope is slated to launch by May 2027. Aboard it is a coronagraph with an inner working angle of 0.15 arcseconds. If this system is viewed face-on, what is the furthest distance from which Roman can still resolve the planet, in parsecs?

<sup>1</sup>Where we approximate the central body as "fixed".

17. **The Three Body Problem.** In *The Three Body Problem* (2008) by Cixin Liu, Trisolaris is a fictional planet orbiting in a chaotic three-star system. Canonically, Trisolaris orbits in the (non-fictional) Alpha Centauri system. In 2016, the existence of an exoplanet inside the habitable zone of Proxima Centauri was confirmed; however, Proxima Centauri is on such a wide orbit that the planet hardly feels the other two stars in the system. In this problem, we will explore a more interesting three body system.

First, let's do a warmup problem...

- (a) [2 pts] Suppose we have an object moving in a central potential, such that its equation of motion  $\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r}$ , where  $\ddot{\vec{r}}$  is acceleration vector,  $M$  is some mass,  $r$  is the distance to the origin, and  $-\hat{r}$  points towards the origin. If this object's orbit has a semi-major axis of  $a$ , what is its period? Give your answer as an expression in terms of  $M$ ,  $a$ , and any relevant constants of nature.  
(*Hint: This is equivalent to a small planet orbiting around a star of mass  $M$ !*)

Now, consider a system of three sunlike (i.e. same mass, radius, luminosity, etc.) stars in the stable configuration shown in Image 29, which we will call I, II, and III. Note that each star synchronously follows an identically-shaped orbit; at all points in the orbit, the three stars form an equilateral triangle (gray dashed lines). The black square in the center denotes the barycenter.

- (b) [1.5 pts] If the three stars are all a distance  $r$  from the center of the system, what is the distance from one star to another?
- (c) [1.5 pts] What is the **net** force (magnitude and direction) that Star I feels from Stars II and III? Give your answer as an expression in terms of  $r$ , the star mass  $M = M_{\odot}$ , and any relevant constants of nature.
- (d) [2 pts] Newton's 2nd law says that Star I will move with equation of motion  $M_{\odot}\ddot{\vec{r}} = \vec{F}$ , where  $\vec{F}$  is the force you computed in part (c). Which of Kepler's laws apply here, **and why**?  
(*Hint: Can you equate Star I's equation of motion to that of a planet orbiting a star of some mass?*)
- (e) [1 pt] Each star's orbit has a semi-major axis of 202.4 AU. What is the period of this orbital system in years?

Trisolaris orbits around Star I in a circular orbit with radius 1 AU. Each of Stars I, II, and III have highly eccentric orbits, with a semi-major axis of length 202.4 AU and an eccentricity of 0.997. Currently, **the star system is at apoastron**, and the inhabitants of Trisolaris are enjoying a period of prosperity. Refer to Image 30, where the orbit of Trisolaris is given by the green dashed circle. Because  $M_{\oplus} \ll M_{\odot}$ , you may assume that Trisolaris does not affect the orbits of the three stars.

Trisolaris has Earth-like properties. That is, it has a mass and radius of  $1 M_{\oplus}$  and  $1 R_{\oplus}$ , respectively. Under the ideal greenhouse model, Trisolaris has an albedo of  $\alpha = 0.3$  and emissivity  $\epsilon = 0.78$ .

- (f) [1 pt] What is the current distance from one star to another? Give your answer in AU.
- (g) [1 pt] Compute the flux that Trisolaris currently receives from Star I. Give your answer as a fraction of the solar constant  $G_{SC}$ .
- (h) [2 pts] Compute the flux that Trisolaris currently receives from Star II. Give your answer as a fraction of the solar constant  $G_{SC}$ .
- (i) [2 pts] From the combined fluxes of Stars I, II, and III, compute the current equilibrium temperature of Trisolaris in Kelvins, ignoring its atmosphere.  
(If you did not answer part (h), use a total flux of  $2 G_{SC}$ .)

- (j) [2 pts] Calculate the surface temperature in Kelvins of Trisolaris, taking into account its atmosphere. Is Trisolaris in the habitable zone?

(If you did not answer part (h), use a total flux of  $2 G_{SC}$ .)

(Recall: The ideal greenhouse model has a surface layer with temperature  $T_s$  and a single atmospheric layer with temperature  $T_a$  that is transparent to shortwave (stellar) radiation but partially opaque to longwave (Trisolaran) thermal radiation. The key energy balance equations  $E_{in} = E_{out}$  are: 1.  $\frac{1}{4}S_0(1 - \alpha) = \epsilon\sigma T_a^4 + (1 - \epsilon)\sigma T_s^4$  and 2.  $\frac{1}{4}S_0(1 - \alpha) + \epsilon\sigma T_a^4 = \sigma T_s^4$ , where  $S_0$  is the incident flux.)

Now, let's fast forward to when **the star system is at periastron** (see Image 31). At this point, the stars are so close together that Trisolaris's is now quite chaotic, and its orbit crosses the barycenter.

- (k) [1 pt] What is the distance from each star to the barycenter?
- (l) [1 pt] Approximate the average total flux that Trisolaris receives from its suns **by placing Trisolaris at the barycenter**. Give your answer in terms of  $G_{SC}$ .
- (m) [2 pts] Compute the equilibrium temperature of Trisolaris at periastron in Kelvins, ignoring its atmosphere. Is Trisolaris still in the habitable zone?
- (If you did not answer part (l), use a total flux of  $5 G_{SC}$ .)

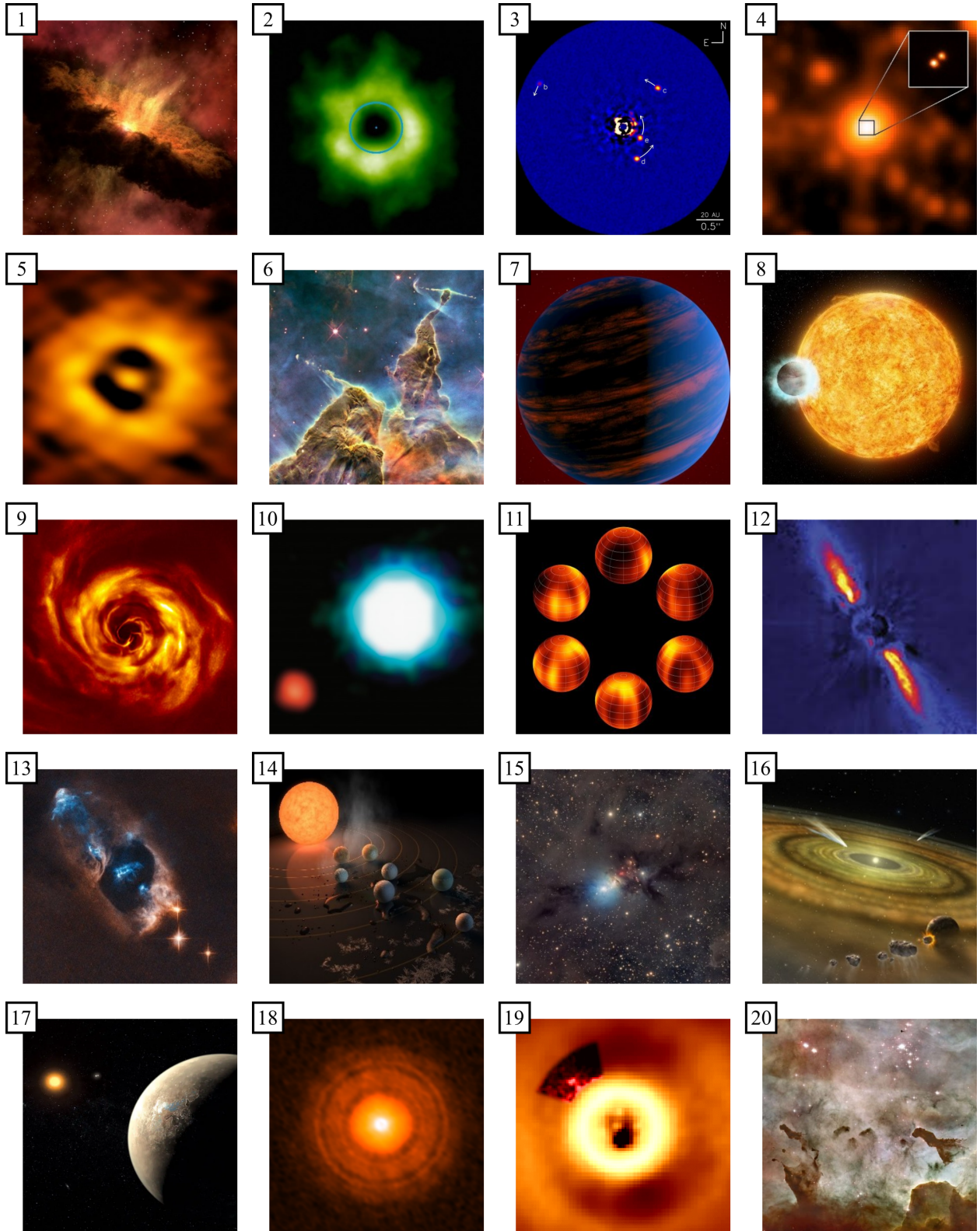
*Looks like Trisolaris is in trouble in a few ten thousand years...*

18. **From Afar.** (*Important Note: Answers to Question 17 are not necessary to answer this question.*)

Now, let's imagine we are on Earth observing the Trisolaran star system, located at a distance of 4.24 light years. Recall from Question 17 that the Trisolaran system is composed of three sunlike stars in the configuration shown in Image 29, and that Trisolaris is an Earthlike planet, orbiting around Star I in a circular orbit of 1 AU. Suppose that from Earth, this system has inclination of  $90^\circ$  (edge-on).

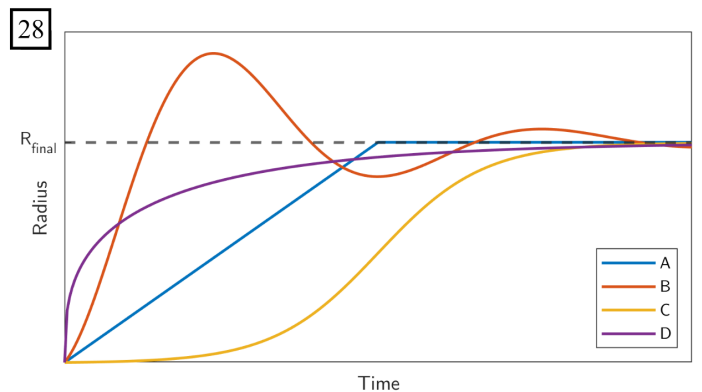
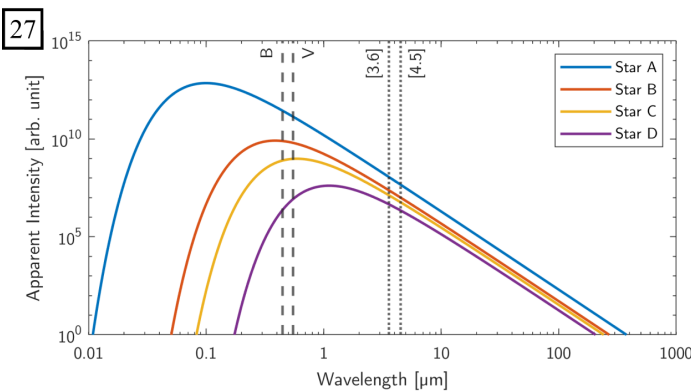
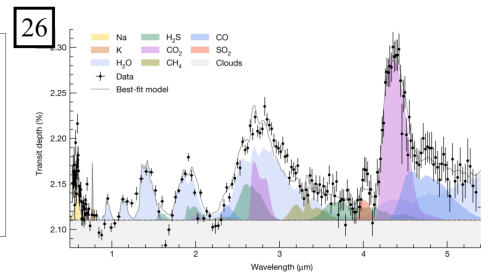
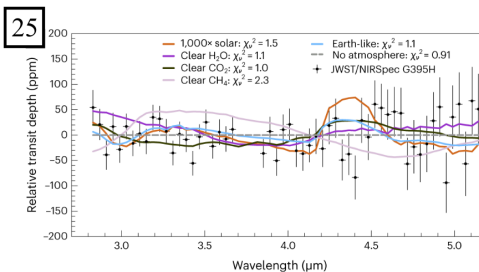
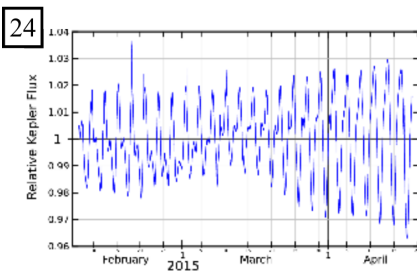
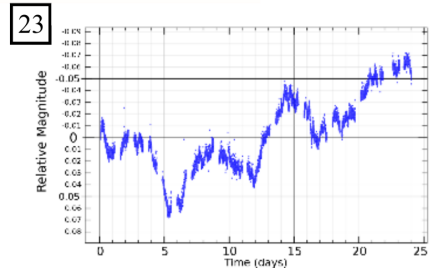
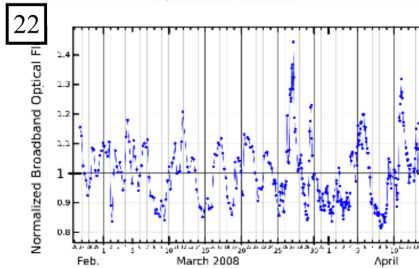
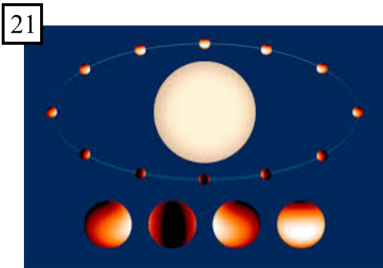
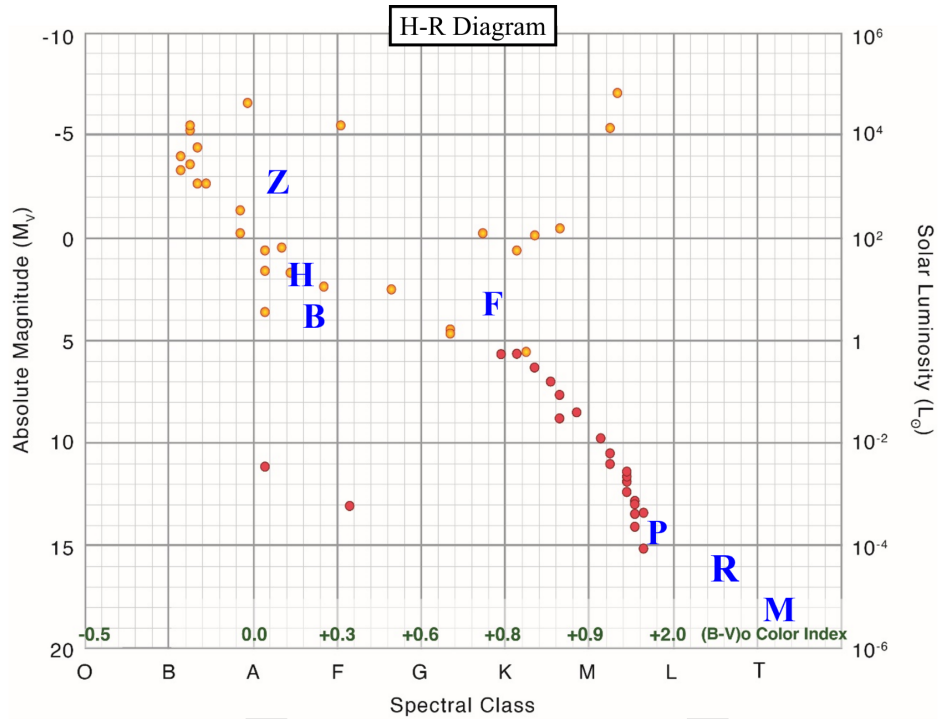
- (a) [1 pt] Compute the parallax of the Trisolaran star system in arcseconds.
- (b) [1.5 pts] Compute the bolometric apparent magnitude of a single star.
- (c) [1.5 pts] Compute the *combined* absolute magnitude of the Trisolaran star system.
- (d) [1.5 pts] Suppose we are looking for exoplanets around this system via the radial velocity method. What is the radial velocity amplitude of Star I caused by the orbit of Trisolaris? Give your answer in m/s.
- (e) [1.5 pts] If we're looking at the H-alpha spectral line ( $\lambda = 656.3$  nm) of Star I, what wavelength resolution do we need to detect Trisolaris? That is, compute the corresponding amplitude in wavelength from your answer to part (d). Give your answer in nm.

### Image/Illustration Set A



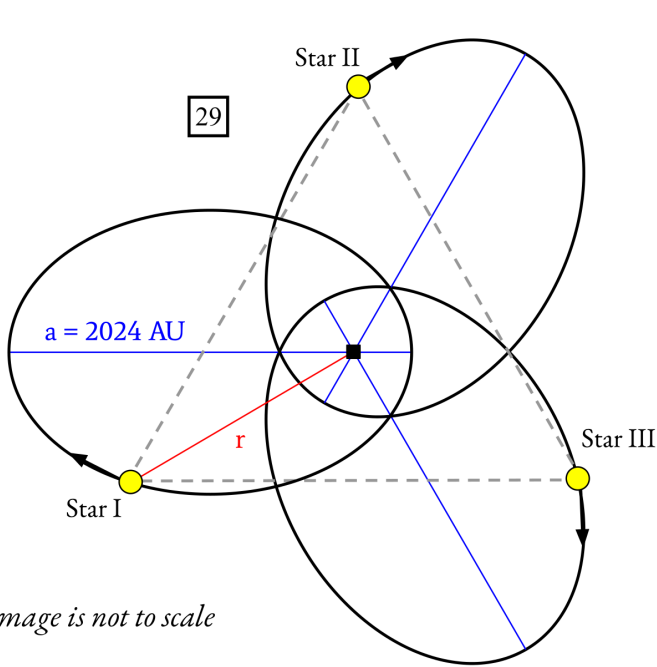


### Image/Illustration Set B

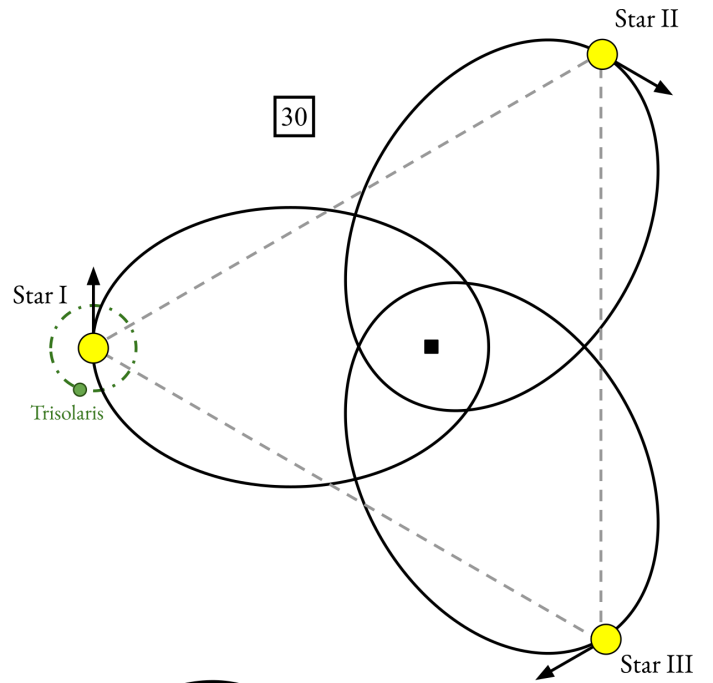




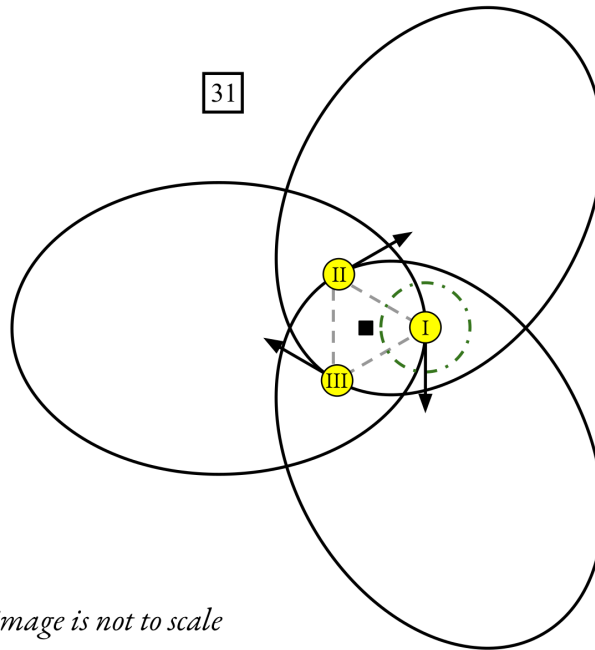
Image/Illustration Set C



*\*\*image is not to scale*



31



*\*\*image is not to scale*

Science Olympiad  
Astronomy C  
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May 25, 2024



Answer Sheet

Team Name and Number: \_\_\_\_\_

Participant Name(s): \_\_\_\_\_

Total Score: \_\_\_\_ / 100

Rank: \_\_\_\_

**Directions:**

- Read the directions on the exam cover.

### Section A (52 points)

1. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

2. (a) \_\_\_\_\_

\_\_\_\_\_

(b) \_\_\_\_\_

3. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

\_\_\_\_\_

4. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

5. (a) \_\_\_\_\_

(b) \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

(c) \_\_\_\_\_

(d) \_\_\_\_\_

6. (a) \_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

7. (a) \_\_\_\_\_

\_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

(d) \_\_\_\_\_

8. (a) \_\_\_\_\_

(b) \_\_\_\_\_

\_\_\_\_\_

9. (a) \_\_\_\_\_

\_\_\_\_\_

(b) \_\_\_\_\_

(c) \_\_\_\_\_

(d) \_\_\_\_\_

10. (a) \_\_\_\_\_

(b) \_\_\_\_\_

11. (a) \_\_\_\_\_

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(b) \_\_\_\_\_

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\_\_\_\_\_

(c) \_\_\_\_\_

12. (a) \_\_\_\_\_

(b) \_\_\_\_\_

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13. (a) \_\_\_\_\_

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(b) \_\_\_\_\_

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(c) \_\_\_\_\_

(d) \_\_\_\_\_

(e) \_\_\_\_\_

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(f) \_\_\_\_\_

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14. (a) \_\_\_\_\_

(b) \_\_\_\_\_

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(c) \_\_\_\_\_

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(d) \_\_\_\_\_

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(e) \_\_\_\_\_

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**Section B (15 points)**

- 15. (a) \_\_\_\_\_  
\_\_\_\_\_
- (b) \_\_\_\_\_  
\_\_\_\_\_
- (c) \_\_\_\_\_  
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- (d) \_\_\_\_\_  
\_\_\_\_\_
- (e) \_\_\_\_\_  
\_\_\_\_\_
- (f) \_\_\_\_\_  
\_\_\_\_\_
- (g) \_\_\_\_\_  
\_\_\_\_\_
- (h) \_\_\_\_\_  
\_\_\_\_\_
- (i) \_\_\_\_\_  
\_\_\_\_\_
- (j) \_\_\_\_\_  
\_\_\_\_\_

### Section C (33 points)

16. (a)

(b)

(c)

(d)

17. (a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

(m)



18. (a)

(b)

(c)

(d)

(e)